Journal of Engineering Physics and Thermophysics, Vol. 81, No. 2, 2008

ON FLUIDIZATION OF POLYDISPERSE GRANULAR MATERIALS

Yu. A. Teplitskii

UDC 66.096.5

A method has been developed for calculating the fractional composition of boiling bed particles liquefied at a given speed of filtration with account for the composition of the supplied granular materials. The relation between the maximum size of particles and their mean diameter and the degree of polydispersity has been investigated.

Fluidization of polydisperse granular materials has many specific features which should be taken into account in hydrodynamic and thermal calculations of boiling-bed apparatuses [1]. The intensity of the process of external and internal (interphase) heat transfer is also determined, apart from other factors, by the composition of the disperse material [2, 3]. The hydrodynamics of such a system strongly depends on the mechanism of momentum transfer from more mobile small fractions to large ones, due to which the latter are liquefied at rates much lower than the initial rates of their fluidization. The fluidization curve of a polydisperse material, as experience shows [2], differs appreciably from the fluidization curve of the bed of one-size particles: the characteristic horizontal portion at $u > U_{\rm ff}$ and the pressure peak upon the transition of the bed to the liquefied state are absent, which does not permit graphical determination of the $u_{\rm ff}$ value by this curve. If the whole of the monodisperse bed at $u < u_{\rm mf}$ is in the stationary state, then in the polydisperse bed at $u < u_{\rm ff}$ a clear separation of particles is observed: small fractions form a boiling bed located over the stationary bed of large fractions. Between these two beds there is a marked boundary which moves towards the gas-distributing lattice with increasing gas velocity and reaches it at $u = u_{\rm ff}$ (moment of the start of boiling of the whole of the polydisperse bed). At such a velocity in the bed there exist gas bubbles of large sizes and a noticeable mixing of particles is observed [2]. This is not observed at $u = u_{\rm mf}$ in the bed of monofraction particles.

The polydisperse mixture of particles is characterized by a discrete distribution: d_i, η_i ($i = 1, ..., N$). The weight fractions of fractions are related by the apparent condition

$$\sum_{i=1}^{N} \eta_i = 1 \,. \tag{1}$$

The arithmetic mean $\langle d \rangle$ and geometric mean $\langle \bar{d} \rangle$ diameters of particles [1, 3] reflecting their composition are introduced:

$$\langle d \rangle = \sum_{i=1}^{N} \eta_i d_i \,, \tag{2}$$

$$\langle \bar{d} \rangle = \left[\sum_{i=1}^{N} \eta_i / d_i \right]^{-1} . \tag{3}$$

Let us determine the degree of polydispersity of the particle mixture by analogy with the dispersion of a discrete random quantity [4]

$$\varphi = \sum_{i=1}^{N} \eta_i \frac{(d_i - \langle d \rangle)^2}{\langle d \rangle^2} \,. \tag{4}$$

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 81, No. 2, pp. 353–357, March–April, 2008. Original article submitted April 28, 2006.

1062-0125/08/8102-0374©2008 Springer Science+Business Media, Inc.

In [5], on the basis of systematic experimental studies of the fluidization of polydisperse mixtures of a wide fractional composition it has been established that:

a) to calculate the rate of complete liquefaction of polydisperse particles u_{ff}, one can use the known Todes formula [6]

$$\mathrm{Re}_{\mathrm{ff}} = \frac{\mathrm{Ar}}{1400 + 5.22\sqrt{\mathrm{Ar}}}, \tag{5}$$

if $\langle d \rangle$ determined by (2) is taken as a particle diameter in $\mathrm{Re}_{\mathrm{ff}}$ and Ar numbers;

b) to calculate the conductive-inductive heat transfer coefficient of a bed with flat surfaces, one can use the dependence obtained on the basis of generalizing a large body of experimental data in beds of monodisperse particles [7]:

$$\mathrm{Nu} = 0.74\overline{\mathrm{Ar}}^{0.1}\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\right)^{0.14}\left(\frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}\right)^{0.24}(1-\varepsilon)^{2/3} + 0.046\overline{\mathrm{Re}}\,\mathrm{Pr}\frac{(1-\varepsilon)^{2/3}}{\varepsilon}, \tag{6}$$

if $\langle \bar{d} \rangle$ has been determined from formula (3) as a particle diameter in $\overline{\mathrm{Ar}}$ and $\overline{\mathrm{Re}}$. As applied to the heat transfer of a bed with finned surfaces, an analogous conclusion has been obtained in [8]. The use of the quantity $\langle \bar{d} \rangle$ as a characteristic size for calculating the processes of external heat transfer has a simple physical meaning: small fractions produce the greatest effect on the quantity $\langle \bar{d} \rangle$ and largely determine the effective thickness of the gas film near the heat transfer surface representing, as is known [9], the main thermal resistance to the heat transfer. This conforms well to the fact that the expression for $\langle \bar{d} \rangle$ follows from the condition of equivalence of the surfaces of polydisperse mixture particles and particles of diameter $\langle \bar{d} \rangle$ [10]. In view of this, it may be concluded that in describing the processes proceeding on the surface of polydisperse particles (heat and mass transfer, friction, etc.) one has to use the quantity $\langle \bar{d} \rangle$. In describing the processes where the determining role is played by the mass (body) forces (onset of complete liquefaction of the mixture), it is necessary to use $\langle d \rangle$. This is confirmed by the use in [9] of the quantity $\langle \bar{d} \rangle$ as a characteristic diameter in the Ergan formula for calculating the resistance of the stationary brown bed which is determined by the surface friction forces. Note that in all these cases there is no need to take into account the degree of polydispersity.

In the practical use of polydisperse granular materials in boiling-bed apparatuses, it is important to know the evolution of the fractional composition of particles injected into the bed, as well as to be able to estimate quantitatively the degree of influence of small fractions on the liquefaction of large ones. In this connection, the aim of the present work was to develop a method for calculating the fractional composition of boiling bed particles on the basis of the analysis of the granular material fed into the bed and elucidate the influence of the material polyfractionality on the maximum size of liquefied particles.

Evolution of the Fractional Composition of the Polydisperse Material Fed into the Apparatus. For the sake of simplicity, we assume that in the system any processes leading to an increase (decrease) in the particle sizes are absent, i.e., purely hydrodynamical causes of the change in the composition of particles injected into the boiling bed at a given speed of filtration (entrainment of small fractions and sedimentation of large ones) are considered. We assume that the composition of the fed material is described by the following discrete distribution: d_i, η_i (i = 1, ..., N). For the definition, we assume $d_i > d_{i+1}$, i.e., the largest fraction has the diameter d_1 and the smallest one d_N.

Change in the composition of the material connected with entrainment. Evidently, particles for whose hovering velocity the condition

$$(u_{\mathrm{t}})_i < u\,, \quad i = m, \ldots, N \tag{7}$$

(m is the number of the largest fraction for which (7) holds) is fulfilled will be carried over with the gas flow. The hovering velocity is calculated by the Todes formula [6]

$$(\mathrm{Re}_{\mathrm{t}})_i = \frac{\mathrm{Ar}_i}{18 + 0.6\sqrt{\mathrm{Ar}_i}}. \tag{8}$$

Then the mixture of particles remaining in the apparatus (but not all of them in the boiling bed!) will be described by the distribution

$$d_i\,,\ \ \eta_i^* \quad (i = 1, ..., m-1)\,,\ \ \eta_i^* = \eta_i \Bigg/ \left(1 - \sum_{j=m}^{N} \eta_j\right). \tag{9}$$

Change in the composition of particles connected with the sedimentation of large fractions. Such a change is possible under the condition $u < u_{\text{ff}}^*$, where u_{ff}^* is determined by formula (5) at $\langle d \rangle = \sum_{i=1}^{m-1} \eta_i^* d_i$. At such a speed of filtration the largest fractions will settle at the bottom of the apparatus and form, over the gas distributor, a stationary blown bed of particles having a marked boundary with the boiling bed of smaller fractions. Below we give a simple method for calculating the fractional composition of fluidized particles based on the use of formulas (2) and (5).

We first calculate the equivalent diameter of the mixture without the largest fraction ($d_1 = d_{\max}$):

$$\langle d \rangle_1 = \sum_{i=2}^{m-1} \eta_i^* d_i / (1 - \eta_1^*)\,. \tag{10}$$

Then we determine by (5) for $d = \langle d \rangle_1$ the rate of complete liquefaction of the new polydisperse mixture (10), $(u_{\text{ff}}^*)_1$. If $u > (u_{\text{ff}}^*)_1$, then the calculation is completed. If $u < (u_{\text{ff}}^*)_1$, then the procedure is repeated — particles with diameter d_2 are removed, and so on. In the general case, calculation is performed until the condition

$$(u_{\text{ff}}^*)_k < u\,, \tag{11}$$

in which $(u_{\text{ff}}^*)_k$ is calculated by (5) for $\langle d \rangle = \langle d \rangle_k$, is met. The equivalent diameter and the fractional composition of liquefied particles are calculated by the formulas

$$\langle d \rangle_k = \sum_{i=k+1}^{m-1} \eta_i^{**} d_i\,,\ \ \eta_i^{**} = \eta_i^* \Bigg/ \left(1 - \sum_{j=1}^{k} \eta_j^*\right). \tag{12}$$

In view of (9) for η_i^{**} we get

$$\eta_i^{**} = \eta_i \Bigg/ \left(1 - \sum_{j=1}^{k} \eta_j - \sum_{j=m}^{N} \eta_j\right). \tag{13}$$

Thus, the fluidized mixture of particles is described by the following distribution:

$$d_i\,,\ \ \eta_i^{**} \quad (i = k+1, ..., m-1)\,, \tag{14}$$

which represents the evolution of the initial composition of particles d_i, η_i (i = 1, ..., N). The large fractions d_1, ..., d_k therewith will settle at the bottom of the apparatus and form a blown stationary bed; the latter, together with the gas-distributing lattice, can be considered as a kind of a gas distributor over which a bed of particles with a composition described by distribution (14) is boiling. The maximum particle size of this bed $d_{\max} = d_{k+1}$. This simple method is important and convenient for practical use in estimating the composition of particles which, when fed into the apparatus, will be carried over with the gas flow (small fractions), settle on the gas-distributing lattice (large fractions), and, consequently, will not participate in the given technological process. Moreover, the proposed method permits calculating the value of the maximum diameter of particles liquefied at a given speed of filtration of the gas needed for practical calculations of the processes of particle processing in the boiling bed.

TABLE 1. Polydisperse Granular Mixtures Used to Obtain Relation (16)

Material	d_i, mm	η_i	$\langle d \rangle$, mm	u_{ff}, m/sec	φ	ρ_s, kg/m^3	$d_{\max}$, mm
Glass balls [3]	4.37	0.507	5.08	1.87	0.02	2650	5.79
	5.79	0.493					
Glass balls [3]	4.37	0.2335	7.56	2.35	0.10	2650	9.85
	5.79	0.2475					
	9.85	0.5190					
Glass balls [3]	4.37	0.1655	9.8	2.70	0.125	2650	12.91
	5.79	0.1515					
	9.85	0.1985					
	12.91	0.4845					
Glass balls [3]	4.37	0.1280	13.63	3.20	0.128	2650	18.56
	9.85	0.1820					
	12.91	0.2700					
	18.56	0.4200					
Aggloporite [2]	<1	0.097	3.43	1.1	0.39	1640	10
	3—1	0.439					
	5—3	0.205					
	7—5	0.163					
	10—7	0.096					
Anthracite [3]	0.457	0.03	0.69	0.23	0.042	1970	0.914
	0.533	0.25					
	0.635	0.23					
	0.762	0.29					
	0.914	0.20					
Sand [10]	0.0625	0.167	0.096	0.0092	0.1	2650	0.162
	0.0875	0.150					
	0.112	0.333					
	0.137	0.167					
	0.162	0.083					

Analysis of the Influence of Small Fractions on the Liquefaction of Large Ones. For such analysis, let us establish the relation of the maximum size of particles to the mean diameter of the polyfraction mixture and the degree of polydispersity. We give the sought relation in the following form:

$$\frac{d_{\max}}{\langle d \rangle} = 1 + A\varphi^n . \tag{15}$$

Formula (15) describes the influence of polyfractionality on $d_{\max}/\langle d \rangle$ and yields at $\varphi = 0$ $d_{\max} = \langle d \rangle = d$ (monofractional particles).

To determine the coefficients A and n, we have analyzed seven particular polydisperse mixtures (see Table 1). Data processing by (15) led to the simple formula

$$\frac{d_{\max}}{\langle d \rangle} - 1 = 3.0\varphi^{0.8} , \quad 0.02 \le \varphi \le 0.39 . \tag{16}$$

Using (16), let us determine the ratio between the rate of onset of fluidization of particles $d_{\max}$ and the rate of complete liquefaction of the mixture of particles with a mean diameter $\langle d \rangle$. For simplicity's sake, consider the case of large particles ($\mathrm{Re}_{\mathrm{ff}} > 1000$). Then the Todes formula (5) for them is of the form

$$\mathrm{Re}_{\mathrm{ff}}=\frac{\sqrt{\mathrm{Ar}}}{5.22}\,. \tag{17}$$

Let us give (17) in a more convenient form

$$\mathrm{Fr}_{\mathrm{ff}}=\frac{u_{\mathrm{ff}}^{2}}{g\,\langle d\rangle}=0.036\,\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\,. \tag{18}$$

To determine the rate of onset of fluidization of particles $d_{\max}$, we have

$$\mathrm{Fr}_{\max}=\frac{(u_{\mathrm{mf}})_{\max}^{2}}{g d_{\max}}=0.036\,\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\,. \tag{19}$$

In view of (16), (18), we obtain the sought rate ratio

$$\frac{(u_{\mathrm{mf}})_{\max}}{u_{\mathrm{ff}}}=\sqrt{1+3.0\varphi^{0.8}}\,. \tag{20}$$

Formula (20) takes into account the influence of small fractions on the liquefaction of large ones and shows that large particles in the composition of polydisperse mixtures can liquefy at a much lower rate (u_{ff}) than their rate of onset of fluidization $(u_{\mathrm{mf}})_{\max}$.

Formula (20) can be inverted and the expression for φ

$$\varphi=0.25\left(((u_{\mathrm{mf}})_{\max}/u_{\mathrm{ff}})^{2}-1\right)^{1.25} \tag{21}$$

can be obtained. Relation (21) makes it possible to select, at given values of the speed of filtration and $(u_{\mathrm{mf}})_{\max}$, an optimal composition of the polydisperse mixture with $u_{\mathrm{ff}} = u$ permitting liquefaction of large particles of diameter $d_{\max}$, whose rate of onset of fluidization $(u_{\mathrm{mf}})_{\max}$ is much higher than u.

NOTATION

$\mathrm{Ar}_i = \frac{g d_i^3}{\nu_{\mathrm{f}}^2}\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}-1\right)$; $\mathrm{Ar} = \frac{g\langle d\rangle^3}{\nu_{\mathrm{f}}^2}\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}-1\right)$; $\overline{\mathrm{Ar}} = \frac{g\langle \bar{d}\rangle^3}{\nu_{\mathrm{f}}^2}\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}-1\right)$, Archimedes numbers; c_{s}, c_{f}, specific heat capacity of particles and gas, respectively, J/(kg·K); d, diameter of particles, m; g, free fall acceleration, m/sec^2; N, number of fractions; $\mathrm{Nu} = \alpha\langle \bar{d}\rangle/\lambda_{\mathrm{f}}$, Nusselt number; Pr, Prandtl number; $\mathrm{Re}_i = u d_i/\nu_{\mathrm{f}}$, $(\mathrm{Re}_{\mathrm{t}})_i = (u_{\mathrm{t}})_i d_i/\nu_{\mathrm{f}}$, $\mathrm{Re}_{\mathrm{ff}} = u_{\mathrm{ff}}\langle d\rangle/\nu_{\mathrm{f}}$; $\overline{\mathrm{Re}} = u\langle d\rangle/\nu_{\mathrm{f}}$, Reynolds numbers; u_{mf}, rate of onset of fluidization (monodisperse particles), m/sec; u_{ff}, rate of complete liquefaction (polydisperse mixture of particles), m/sec; u, speed of filtration, m/sec; α, heat transfer coefficient, W/(m^2·K); ε, porosity; η_i, weight fraction of particles of diameter d_i; λ_{f}, heat conductivity coefficient of the gas, W/(m·K); ν_{f}, kinematic viscosity of the gas, m^2/sec; ρ_{s} and ρ_{f}, particle and gas densities, respectively; kg/m^3; φ, degree of polydispersity. Subscripts: f, gas; ff, complete liquefaction; i, fraction number; k, number of non-liquefied large fractions; mf, onset of fluidization; max, maximum; s, particles; t, hovering condition of a single particle.

REFERENCES

1. I. M. Razumov, *Fluidization and Pneumatic Transport of Loose Materials* [in Russian], Khimiya, Moscow (1972).

2. V. A. Borodulya, V. L. Ganzha, Yu. S. Teplitskii, and Yu. G. Epanov, *Fluidization of Polydispersed Layers of Large Particles* [in Russian], Preprint No.4 of the A. V. Luikov Heat and Mass Transfer Institute, Minsk (1984).
3. M. Leva, *Fluidization* [in Russian], Gos. Nauch.-Tekh. Izd. Neftyanoi i Gorno-Toplivnoi Lit., Moscow (1961).
4. G. Korn and T. Korn, *Handbook of Mathematics* [Russian translation], Nauka, Moscow (1974).
5. Yu. S. Teplitskii, *Hydrodynamics and Heat and Mass Transfer in a Free and Retarded Fluidized Bed*, Author's Abstract of Doctoral Dissertation (in Engineering), Novosibirsk (1991).
6. O. M. Todes and O. B. Tsitovich, *Apparatuses with a Fluidized Granular Bed* [in Russian], Khimiya, Leningrad (1981).
7. V. A. Borodulya, Yu. S. Teplitsky, I. I. Markevich, A. F. Hassan, and T. P. Yeromenko, Heat transfer between a surface and a fluidized bed: Consideration of pressure and temperature effects, *Int. J. Heat Mass Transfer*, **34**, No. 1, 47–53 (1991).
8. A. F. Hassan, *Heat and Mass Transfer to Smooth and Ribbed Surfaces in the Superficial Volume of Fluidized Beds of Mono- and Polydispersed Material as Applied to Furnace Facilities*, Author's Abstract of Candidate's Dissertation (in Engineering), Minsk (1992).
9. A. P. Baskakov (Ed.), *Processes of Heat and Mass Transfer in a Fluidized Bed* [in Russian], Metallurgiya, Moscow (1976).
10. D. Kunii and O. Levenshpiel, *Industrial Fluidization* [Russian translation], Khimiya, Moscow (1978).